Vanishing Loss Effect on the Effective *ac* Conductivity behavior for 2D Composite Metal-Dielectric Films At The Percolation Threshold

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Abstract

We study the imaginary part of the effective ac conductivity as well as its distribution probability for vanishing losses in 2D composites. This investigation showed that the effective medium theory provides only informations about the average conductivity, while its fluctuations which correspond to the field energy in this limit are neglected by this theory.

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Introduction

Although metal-dielectric composites were extensively investigated during the last decade, some cases where the local field and conductivity fluctuations are strong remain not understood [1, 2]. The effective medium theory [3] relates at the percolation threshold, the effective conductivity to the metal and dielectric conductivities by

$$\sigma_{eff} = \sqrt{\sigma_m \sigma_d} \tag{1}$$

The indices m, d and eff stand respectively for the metal, dielectric and effective medium. However, this equation is not satisfied for vanishing loss, where it expects the effective ac conductivity to be real while numerical studies showed the real part to be vanishing [2]. Brouers et al. [4] explained the field behavior in this limiting case as an energy storage of the electromagnetic wave between dielectric and superconducting component. In the same sens, we have found the local field to be localized in this limit [5]. This localization is enhanced by the dissipation for finite losses.

However, no work (to the best of our knowldge) has been devoted to the imaginary part of the effective conductivity in this limit of the loss. This is the aim of the present work where we study the loss effect on the imaginary part of the effective conductivity as well as its distribution in its vanishing limit for a 2D metal-dielectric network.

Model description

The metal-dielectric composites can be modelled as an RLC network where the metallic grains represent the RL branches while dielectric grains correspond to the capacitance C [6]. The frequency dependent conductivities behave then respectively for the metal and dielectric components as

$$\sigma_{m,d} = \frac{-id\omega\epsilon_{m,d}}{4\pi} \tag{2}$$

$$\sigma_m = \frac{1}{iL\omega + R} \tag{3}$$

and

$$\sigma_d = iC\omega \tag{4}$$

R being the loss in the metal. We limit ourselves in this paper to the characteristic frequency ω_{res} where $|\sigma_m| = |\sigma_d|$ for vanishing loss, i.e. $L\omega_{res} = 1/C\omega_{res}$. The inductance L and capacitance C being constant near the characteristic frequency, we can take without loss of generality $L = C = \omega_{res} = 1$. For any frequency close to ω_{res} the frequency ω in these equations is expressed as ω/ω_{res}. Although this form simplifies the calculations, we

can easily determine the true parameters of the dielectric constants of the two components [5].

In order to determine the effective conductivity of the resulting RLC network, we solve exactly the sets of Kirchoff equations of the RLC network by handling the corresponding large matrices by blocs of small matrices due to their sparce shape and arrangement in the diagonal region (see [7]).

Results and discussion

In a 2D square lattice, the percolation threshold is reached when the concentration of the metallic component is $p = 0.5$ [8]. For the rest of the paper, we restrict ourselves to this concentration. We then consider an RLC lattice of 256x256 branches in order to ensure a statistically stable system. We have previously found the real part of the effective conductivity to vanish for a vanishing loss [5]. This result does not agree with the predictions of the effective medium theory (given by Eq. 1) where the imaginary part is expected to vanish while the real one should be finite in this limit. However, in the case of very small losses, the local field strongly fluctuates [1] and the distribution of real part of the effective conductivity is Poissonian and therefore, not characterized by its averaged value [2]. Such fluctuations are not taken into account by the effective medium theory which determines only the average effective conductivity. First, we will study the effect of vanishing loss on the imaginary conductivity (which was not examined in the previous works [2, 5]).

In figure 1, we show the loss dependance of the imaginary part of the effective conductivity for two sample realizations. The main feature in this figure is a transition behavior at around $R = 10^{-5}$ Ohm from a vanishing conductivity for large losses (expected from Eq.1) to finite one for very small losses where this conductivity saturates. This transition region (which separates the two different statistical behaviors for small and large losses) was also observed in the behavior of the real conductivity [2]. We see also from Fig.1 that the imaginary conductivity can have either positive or negative values depending on the sample realization. Therefore the system should strongly fluctuate between metallic (inductive) and dielectric (capacitive) behavior in the limit of very small losses. This leads us to examine the statistical properties of the imaginary effective conductivity.

These statistical properties are clearly shown in figure 2 for two different sizes for comparison ($150x150$ and 256x256) and for 450 realizations with $R = 10^{-9}$ Ohm, where the distribution of the imaginary effective conductivity is a symmetric centered at 0. The fluctuations seem to decrease much slowly with size than the square root of the size (which

is characteristics of normal distributions). Therefore this distribution may have long tails like levy distributions [9]. These results agree with the prediction of Eq. 1 regarding the average imaginary conductivity, but in this case the main information should be provided by its fluctuations and not the average since the complex conductivity vanishes. Indeed, since the real conductivity was found to vanish (its distribution tends to a delat-peak at 0 when the system is very large) as well as the average imaginary conductivity (while the components conductance is finite). It seems that their fluctuations contain informations on the field energy in agreement with the previous arguments [2, 4, 5].

Conclusion

In this article we have studied the imaginary part of the effective conductivity in the limit of vanishing loss. We found it to vanish in avergage but with strong fluctuations. These results show that the average conductivity loses the informations about the field energy which can be provided by the conductivity fluctuations.

Acknowledgements

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Fig.1 The loss dependance of the imaginary part of the effective conductance for two different realization samples.

Fig.2 The distribution of the imaginary part of the effective conductance for 450 samples of two different sizes, the loss is 10^{-9}.
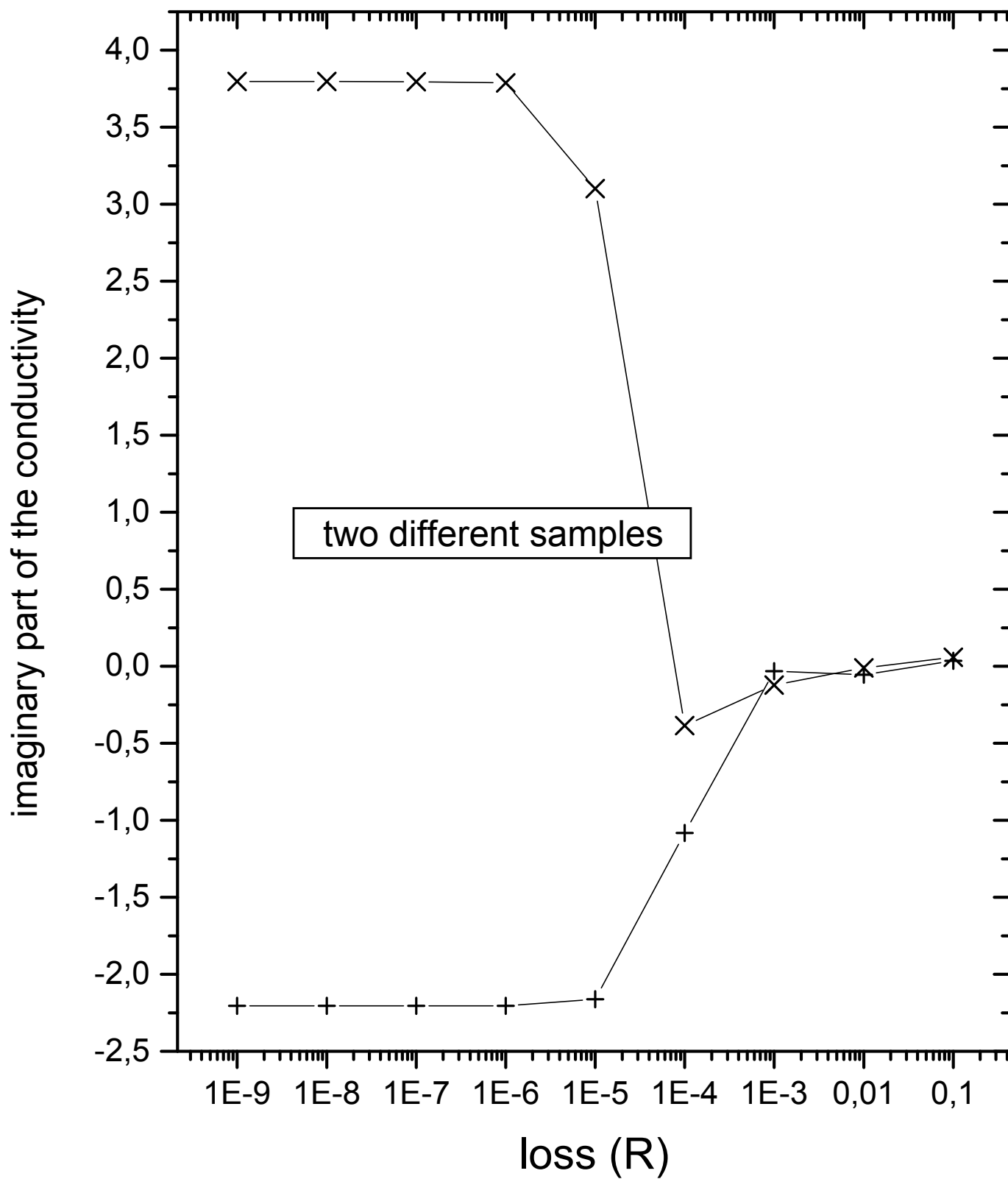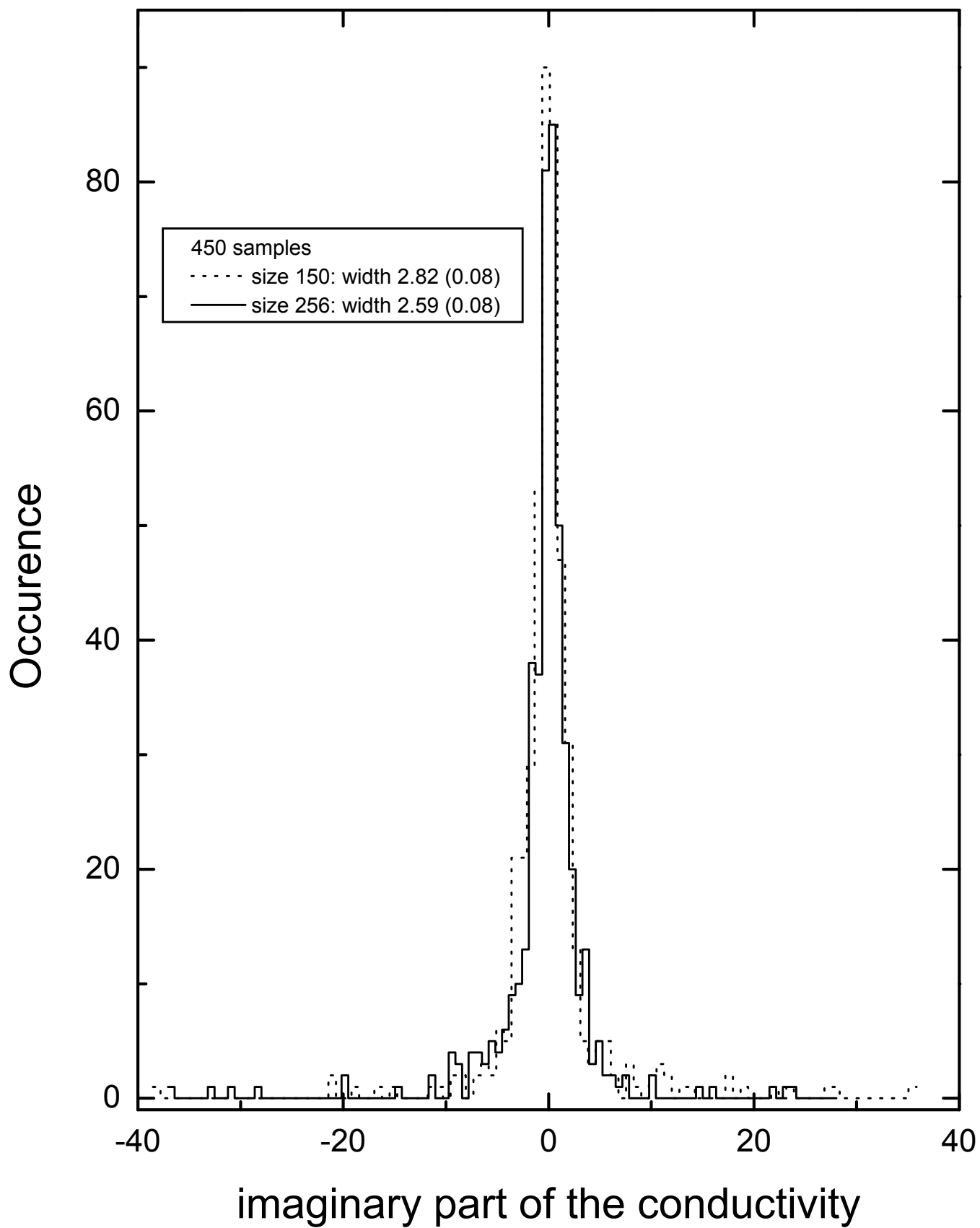
Figure 1

Figure 2